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## Generic Gravitational Corrections to Gauge Couplings in SUSY $SU(5)$ GUTs

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### Abstract

We study non-universal corrections to the gauge couplings due to higher dimensional operators in supersymmetric  $SU(5)$  grand unified theories. The corrections are, in general, parametrized by three components originating from **24**, **75** and **200** representations. We consider the prediction of  $\alpha_3(M_Z)$  along each **24**, **75** and **200** direction, and their linear combinations. The magnitude of GUT scale and its effects on proton decay are discussed. Non-SUSY case is also examined.

1. The prediction of  $\alpha_3(M_Z)$  from the precision data  $\alpha$  and  $\sin\theta_W$  is one of the strong motivations for supersymmetric grand unified theories (SUSY GUTs) [1]. On the analysis of gauge coupling constants, several corrections have been considered, e.g. threshold corrections [2, 3] due to superparticles at the weak scale and heavy particles around the GUT scale. In addition, non-renormalizable interactions in gauge kinetic terms can give corrections suppressed by the reduced Planck mass  $M$  as an effect of quantum gravity [4] \*. We call them gravitational corrections. The gravitational corrections are not universal but proportional to group theoretical factors. If the  $F$ -component of the Higgs field has a non-vanishing vacuum expectation value (VEV), gauginos also receive a non-universal correction to their masses [6].

In the SUSY  $SU(5)$  GUT, gravitational corrections are parametrized by three components originating from **24**, **75** and **200** representations because these (elementary and/or composite) fields can couple to the gauge multiplets in gauge kinetic terms. Non-zero VEVs of  $F$ -component of these fields lead to proper types of non-universal gaugino masses. Recently, several phenomenological aspects of models with such non-universal gaugino masses have been studied and interesting difference among models have been shown [7] †. In our previous analysis, it is assumed that non-universal corrections to gauge couplings  $\alpha_i$  ( $i = 1, 2, 3$ ) are small enough for the  $SU(5)$  breaking scale  $M_U$  to be the ordinary unification scale  $M_X = 2.0 \times 10^{16}\text{GeV}$ .

In this paper, we study gravitational corrections to gauge couplings based on the SUSY  $SU(5)$  GUT. We consider the prediction of  $\alpha_3(M_Z)$  along each **24**, **75** and **200** direction, and their linear combination. We discuss which value is allowed as  $M_U$  in the presence of gravitational corrections and its phenomenological implication. The non-SUSY case is also examined.

The gauge kinetic function is given by

$$\begin{aligned}\mathcal{L}_{g.k.} &= \sum_{\alpha,\beta} \int d^2\theta f_{\alpha\beta}(\Phi^I) W^\alpha W^\beta + H.c. \\ &= -\frac{1}{4} \sum_{\alpha,\beta} \text{Re} f_{\alpha\beta}(\phi^I) F_{\mu\nu}^\alpha F^{\beta\mu\nu}\end{aligned}$$

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\*Such corrections are important for the gauge coupling unification with extra dimensions, too [5].

† See also Ref. [8].

$$+ \sum_{\alpha, \beta, \alpha', \beta'} \sum_I F_{\alpha' \beta'}^I \frac{\partial f_{\alpha \beta}(\phi^I)}{\partial \phi_{\alpha' \beta'}^I} \lambda^\alpha \lambda^\beta + H.c. + \dots \quad (1)$$

where  $\alpha, \beta$  are indices related to gauge generators,  $\Phi^I$ 's are chiral superfields and  $\lambda^\alpha$ 's are the  $SU(5)$  gaugino fields. The scalar and  $F$ -components of  $\Phi^I$  are denoted by  $\phi^I$  and  $F^I$ , respectively. The gauge multiplet is in the adjoint representation and the symmetric product of  $\mathbf{24} \times \mathbf{24}$  is decomposed as

$$(\mathbf{24} \times \mathbf{24})_s = \mathbf{1} + \mathbf{24} + \mathbf{75} + \mathbf{200}. \quad (2)$$

Hence the gauge kinetic function  $f_{\alpha \beta}(\Phi^I)$  is also decomposed as

$$f_{\alpha \beta}(\Phi^I) = \sum_R f_{\alpha \beta}^R(\Phi^I) \quad (3)$$

where  $f_{\alpha \beta}^R(\Phi^I)$  is a part of gauge kinetic functions which transforms as  $R$ -representation ( $R = \mathbf{1}, \mathbf{24}, \mathbf{75}, \mathbf{200}$ ).

After a breakdown of  $SU(5)$  at  $M_U$ , a boundary condition (BC) of  $\alpha_i$  is given by

$$\alpha_i^{-1}(M_U) = \alpha_U^{-1}(1 + C_i) \quad (4)$$

where  $C_i$ 's are non-universal factors which parametrize generic gravitational corrections such as <sup>‡</sup>

$$\begin{aligned} (C_1, C_2, C_3) &= \frac{x_{\mathbf{24}}}{2\sqrt{15}}(-1, -3, 2) + \frac{x_{\mathbf{75}}}{6}(-5, 3, 1) + \frac{x_{\mathbf{200}}}{2\sqrt{21}}(10, 2, 1) \\ &= x'_{\mathbf{24}}(-1, -3, 2) + x'_{\mathbf{75}}(-5, 3, 1) + x'_{\mathbf{200}}(10, 2, 1). \end{aligned} \quad (5)$$

Here  $x_R$ 's are model-dependent quantities including the VEV of Higgs fields, and their order is supposed to be  $O(M_U/M)$  or less.

2. Let us predict  $\alpha_3(M_Z)$  using the experimental values  $\alpha_1^{-1}(M_Z) = 59.98$  and  $\alpha_2^{-1}(M_Z) = 29.57$  based on the assumption that the minimal supersymmetric standard model (MSSM) holds on below  $M_U$  and the SUSY  $SU(5)$  GUT is realized above  $M_U$ .

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<sup>‡</sup>The factors  $1/2\sqrt{15}$ ,  $1/6$  and  $1/2\sqrt{21}$  come from that the normalization  $Tr(T^a T^b) = \delta^{ab}/2$  of the  $5 \times 5$ ,  $10 \times 10$  and  $15 \times 15$  matrices representing  $\mathbf{24}$ ,  $\mathbf{75}$  and  $\mathbf{200}$ .

In the case without gravitational corrections, the following value is obtained

$$\alpha_3^{(0)}(M_Z) = 0.127 \quad (6)$$

based on solutions of one-loop renormalization group (RG) equations with a common SUSY threshold  $m_{SUSY} = M_Z$

$$\alpha_i^{-1}(M_Z) = \alpha_U^{-1} + \frac{b_i}{2\pi} \ln \frac{M_U}{M_Z} \quad (7)$$

where  $(b_1, b_2, b_3) = (33/5, 1, -3)$ . The experimental value of  $\alpha_3(M_Z)$  is [9]

$$\alpha_3(M_Z) = 0.119 \pm 0.002. \quad (8)$$

There are several possibilities to explain the difference between values (6) and (8), e.g., threshold corrections due to superparticles at the weak scale, heavy particles around the GUT scale and gravitational corrections. Here we neglect non-universal threshold corrections and pay attention to gravitational corrections by the **24**, **75** or **200** Higgs field. Before numerical calculations of two-loop RG equations, we estimate the magnitude of  $x_R$  using analytical results of one-loop RG equations with BC (4). For small  $x_R$ ,  $\alpha_3(M_Z)$  receives the following correction,

$$\alpha_3(M_Z) = \alpha_3^{(0)}(M_Z) + \sum_i a_i C_i, \quad (9)$$

where  $a_1 = -0.28$ ,  $a_2 = 0.67$  and  $a_3 = -0.39$ . Using Eq. (5), allowed regions for  $x_R$  are estimated as

$$\begin{aligned} 0.019 &\lesssim x_{\mathbf{24}} \lesssim 0.031, & -0.020 &\lesssim x_{\mathbf{75}} \lesssim -0.012, \\ 0.030 &\lesssim x_{\mathbf{200}} \lesssim 0.050 \end{aligned} \quad (10)$$

for each contribution.

Now let us study corrections solving the two-loop RG equations numerically. We take the top quark mass  $m_t = 175$  GeV and  $\tan \beta = 3$ , and assume the universal SUSY threshold  $m_{SUSY} = 1$  TeV. The three curves in Fig. 1 correspond to predictions of  $\alpha_3(M_Z)$  along the pure **24**, **75** and **200** directions, respectively. Here we use a notation  $x$  instead of  $x_R$ . Fig. 2 shows the magnitude of GUT scale for each case. Thus we obtain a good agreement with the experiment in the region with  $|x| \lesssim O(0.01)$  for the three pure

directions. This value is consistent with  $x_R \lesssim O(M_U/M)$ . For small  $x_R$ ,  $a_i$ 's in eq.(9) are obtained as  $a_1 = -0.27$ ,  $a_2 = 0.61$  and  $a_3 = -0.36$  at the two-loop level. In addition,  $\alpha_3(M_Z)$  receives the SUSY threshold correction,  $0.0039 \times (m_{SUSY}/1 \text{ TeV})$ .

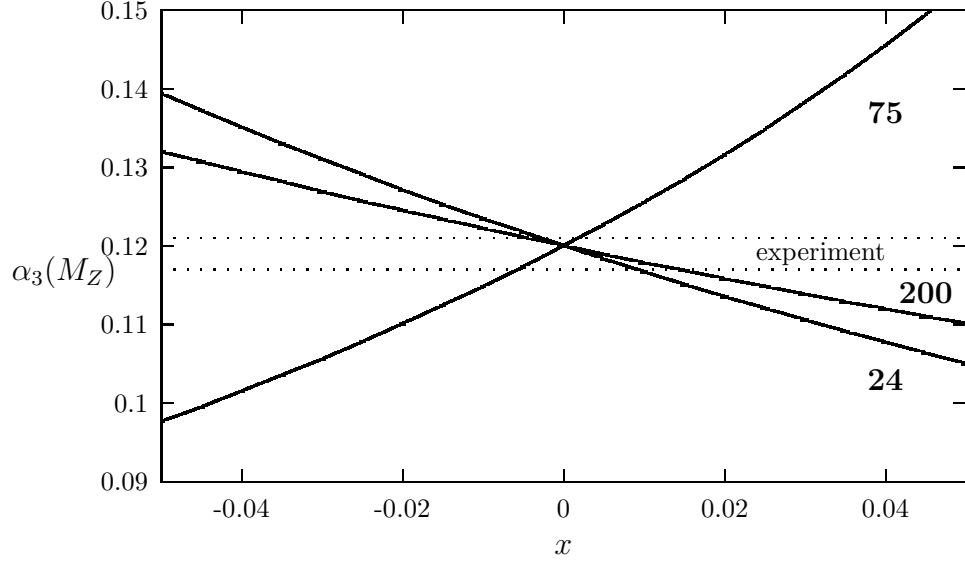


Fig.1:  $\alpha_3(M_Z)$  along the **24**, **75** and **200** directions.

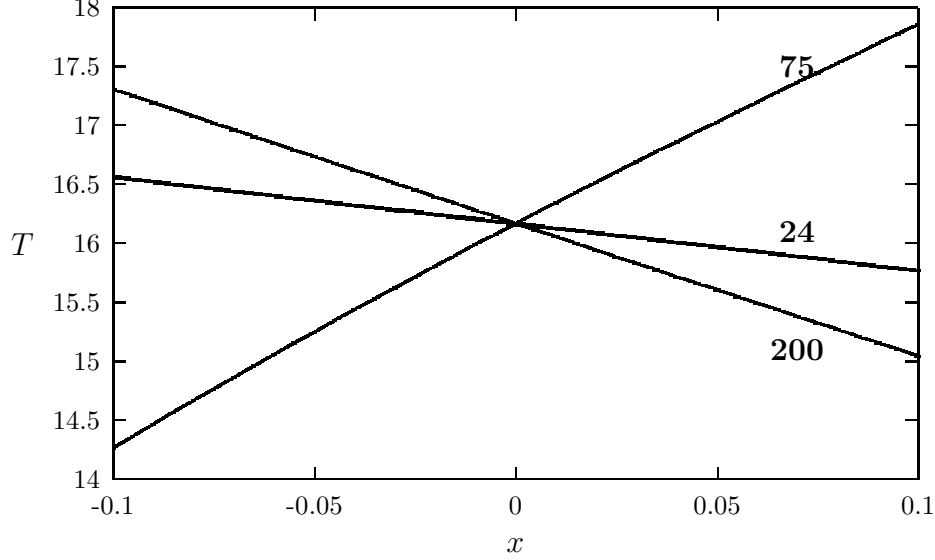


Fig.2: The GUT scale along the **24**, **75** and **200** directions, where  $T$  denotes  $T = \log_{10} M_U$  [GeV].

We give comments on non-universal SUSY threshold corrections. Under the assumption that the dominant contribution to gaugino masses comes from the VEV of  $F$ -component of **24**, **75** or **200** Higgs field, the ratio of gaugino mass magnitudes at the weak scale is given by [6, 7]

$$\begin{aligned}
 M_1 : M_2 : M_3 &= 0.4 : 0.8 : 2.9 & (R = \mathbf{1}), \\
 &= 0.2 : 1.2 : 2.9 & (R = \mathbf{24}), \\
 &= 2.1 : 2.5 : 2.9 & (R = \mathbf{75}), \\
 &= 4.1 : 1.6 : 2.9 & (R = \mathbf{200}).
 \end{aligned}$$

This difference among gaugino masses leads to a small correction compared with the universal SUSY threshold. For example,  $\alpha_3^{(0)}(M_Z)$  is raised by 0.001 for  $R = \mathbf{24}$ . Here we have assumed all soft scalar masses in the MSSM are equal to  $M_3$ . Similarly, the case with **75** Higgs condensation leads to a tiny correction compared with the other two.

Detailed analysis shows that SUSY threshold corrections due to scalar masses could lead to sizable corrections [2]. It is possible to deviate from  $|x| \lesssim O(0.01)$  with a good agreement with the experimental value even in

each pure direction case, although the width of the good parameter region  $\Delta x$  would be as narrow as those in Fig.1, i.e.  $\Delta x = O(0.01)$ .

The GUT scale threshold corrections due to heavy particles are also important to the precise prediction of  $\alpha_3(M_Z)$ . Since they depend on details of a GUT model, it would be difficult to derive model-independent predictions. We will discuss a model with **24** and **75** later.

3. We find that  $|x| \lesssim O(0.01)$  and  $M_U = 10^{16.1 \sim 16.2}$  GeV for the three pure directions without sizable non-universal threshold corrections. Next let us explore a parameter region with a higher breaking scale. It is expected that a higher GUT scale is realized by some linear combination of the **24**, **75** and **200** directions as we see from Figs.1 and 2. Hereafter we consider only contributions from **24** and **75** Higgs fields for simplicity.

First we estimate which value of  $M_U$  is allowed in the presence of gravitational corrections based on one-loop RG analysis. By using solutions of RG equations with the BC (4), the formula of  $M_U$  is given by

$$M_U = M_Z \cdot \exp \left( \frac{2\pi((\alpha_1^{-1} + \alpha_2^{-1} + 2\alpha_3^{-1})(M_Z) - 4\alpha_U^{-1})}{b_1 + b_2 + 2b_3} \right) \quad (11)$$

independent of  $x'_{\mathbf{24}}$  and  $x'_{\mathbf{75}}$ . The  $x'_R$  ( $R = \mathbf{24}, \mathbf{75}$ ) are written in terms of  $\alpha_U$  and  $M_U$  by

$$x'_R = \alpha_U \sum_{i=1}^3 K_R^i (\alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{M_U}{M_Z}) \quad (12)$$

where the matrix  $K_R^i$  is given by

$$K_R^i = \begin{pmatrix} -1/18 & -1/6 & 2/9 \\ -5/36 & 1/12 & 1/18 \end{pmatrix}. \quad (13)$$

Typical values of  $\alpha_U^{-1}$ ,  $x'_{\mathbf{24}}$  and  $x'_{\mathbf{75}}$  are given in Table 1. Here we use  $\alpha_3^{-1}(M_Z) = 8.40$ . This result suggests that the GUT symmetry can be broken down to the SM one  $G_{SM}$  anywhere between  $M_X$  and  $M$ .

Next we solve two-loop RG equations numerically with a common SUSY threshold  $m_{SUSY} = 1\text{TeV}$  and calculate corrections for the linear combination of the **24** and **75** directions, i.e.,  $\langle \mathbf{75} \rangle \cos \theta + \langle \mathbf{24} \rangle \sin \theta$ . Here GUT scale threshold corrections are not considered for simplicity. Fig. 3 shows  $\alpha_3(M_Z)$

Table 1: SUSY case

| $M_U$ (GeV)        | $\alpha_U^{-1}$ | $x'_{\mathbf{24}}$ | $x'_{\mathbf{75}}$ |
|--------------------|-----------------|--------------------|--------------------|
| $2 \times 10^{18}$ | 24.19           | 0.0322             | 0.0235             |
| $2 \times 10^{17}$ | 24.34           | 0.0139             | 0.0083             |
| $2 \times 10^{16}$ | 24.48           | -0.0041            | -0.0067            |

against  $\theta/\pi$  for  $x = 0.01, 0.05$  and  $0.1$  where we set  $x = x_{\mathbf{24}} = x_{\mathbf{75}}$  and Fig. 4 shows  $M_U$  against  $\theta/\pi$ . The correction to  $\alpha_3(M_Z)$  is very small at  $\tan \theta \cong 1.4$  because of a cancellation between contributions from  $\mathbf{24}$  and  $\mathbf{75}$ . The points with  $\tan \theta = 1.4$  are denoted by the dotted vertical lines in Fig. 4. For  $\sin \theta > 0$ ,  $M_U$  increases as  $x$  does at  $\tan \theta \cong 1.4$ , although  $\alpha_3(M_Z)$  does not change. For example,  $x = 0.01, 0.05$  and  $0.1$  correspond to  $M_U = 10^{16.2}$ ,  $M_U = 10^{16.5}$  and  $M_U = 10^{16.9}$  [GeV] at  $\tan \theta = 1.4$  and  $\sin \theta > 0$ , while these values of  $x$  correspond to  $M_U = 10^{16.1}$ ,  $M_U = 10^{15.9}$  and  $M_U = 10^{15.7}$  [GeV] at  $\tan \theta = 1.4$  and  $\sin \theta < 0$ . Such a relatively lower GUT scale is not realized naturally for  $x > O(0.01)$  because the magnitude of  $x$  is expected to be equal to or smaller than  $O(M_U/M)$ .

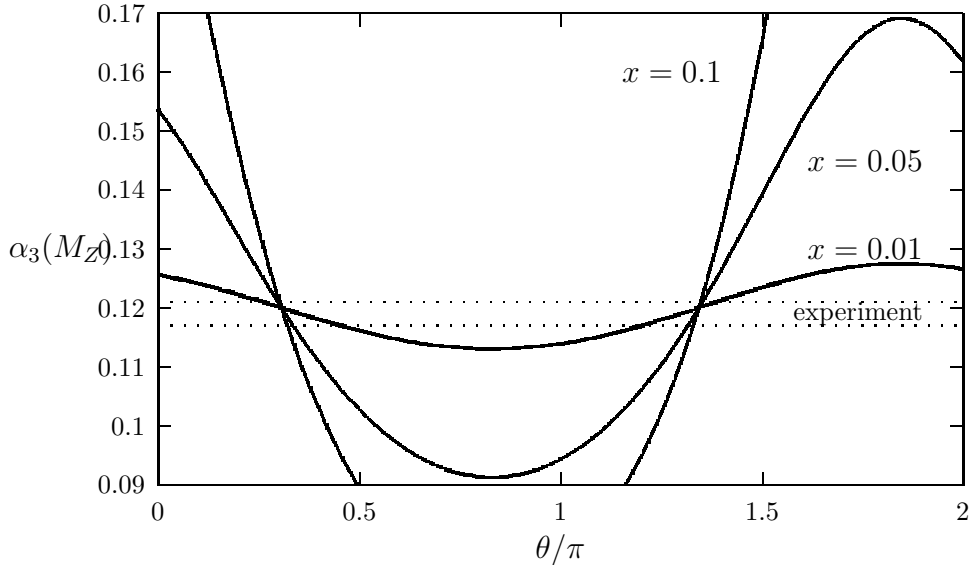


Fig.3:  $\alpha_3(M_Z)$  along linear combinations of  $\mathbf{24}$  and  $\mathbf{75}$  directions.



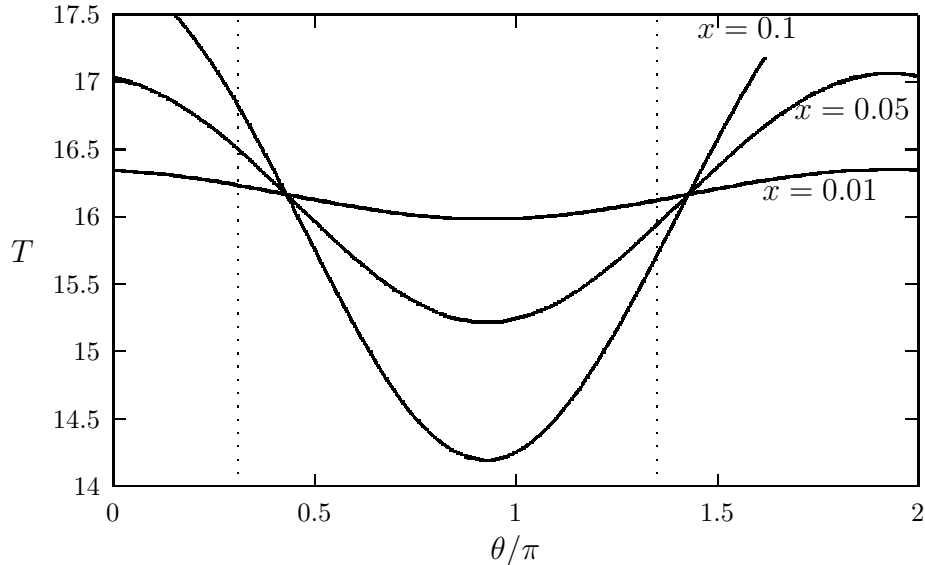


Fig.4: The GUT scale along linear combinations of **24** and **75** directions, where  $T$  denotes  $T = \log_{10} M_X$  [GeV].

Similarly we can discuss the case with generic linear combination including a contribution from the **200** Higgs field. However, the model with the fundamental **200** Higgs particle has so large  $\beta$ -function coefficient that the  $SU(5)$  gauge coupling blows up near to the GUT scale. For example, in a model with the fundamental **200** and **24** Higgs particle and the minimal matter content, the blowing-up energy scale  $M_Y$  is obtained  $M_Y/M_U = 10^{0.74} = 5.5$ . Thus, this model is not connected perturbatively with a theory at  $M$ . There is a possibility that the **200** Higgs field is a composite one made from fields with smaller representations.

4. Finally we discuss proton decay in the presence of gravitational corrections. Here our purpose is to show qualitative features, how much gravitational corrections are important for discussions of proton decay. Thus, we use only one-loop RG equations. In order to carry out a precise analysis, it is necessary to consider two-loop effects, because two-loop effects of RG flows can be comparable to gravitational corrections.

For simplicity, we assume that particle contents of SUSY  $SU(5)$  GUT are  $SU(5)$  gauge multiplet **24**, Higgs multiplets **24** and **75**, and matter multiplets  $N_g(\bar{\mathbf{5}} + \mathbf{10})$ ,  $\mathbf{5} + \bar{\mathbf{5}}$  and extra matter multiplets. Note that, in our usage,

Higgs doublets in the MSSM belong to  $\mathbf{5} + \bar{\mathbf{5}}$  in matter multiplets and we assume that all extra matter multiplets acquire heavy masses much bigger than  $m_{SUSY}$ . The gauge symmetry  $SU(5)$  is assumed to be broken down to  $G_{SM}$  by a combination of VEVs of Higgs bosons  $\mathbf{24}$  and  $\mathbf{75}$ . In this case, would-be Nambu-Goldstone multiplets are a combination of  $(\mathbf{3}, \mathbf{2})$  in  $\mathbf{24}$  and  $\mathbf{75}$  and a combination of  $(\bar{\mathbf{3}}, \mathbf{2})$  in  $\mathbf{24}$  and  $\mathbf{75}$ .

Following the procedure in Ref.[3], we get the following relations at one-loop level,

$$\begin{aligned} & (3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(M_Z) + 12\alpha_U^{-1}(x'_{\mathbf{24}} - x'_{\mathbf{75}}) \\ &= \frac{1}{2\pi} \left( \frac{12}{5} \ln \frac{\hat{M}_C}{M_Z} - 2 \ln \frac{m_{SUSY}}{M_Z} - \frac{12}{5} \Delta_1 \right), \end{aligned} \quad (14)$$

$$\begin{aligned} & (5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(M_Z) + 36\alpha_U^{-1}x'_{\mathbf{75}} \\ &= \frac{1}{2\pi} \left( 36 \ln \frac{\hat{M}_U}{M_Z} + 8 \ln \frac{m_{SUSY}}{M_Z} + 36\Delta_2 \right) \end{aligned} \quad (15)$$

where we use solutions of RG equations of gauge couplings including a universal SUSY threshold and a GUT scale threshold correction. Here  $\hat{M}_C$  and  $\hat{M}_U$  are an effective colored Higgs mass and GUT scale, respectively. For example, in the minimal model  $\hat{M}_C$  is the colored Higgs mass  $M_{H_C}$  itself. The effective GUT scale is given by

$$\hat{M}_U = \left( \frac{M_V^2 M_{\mathbf{24}} M_{\mathbf{75}}}{M'} \right)^{1/3} \quad (16)$$

where  $M_V$  is  $X, Y$  gauge boson mass,  $M_{\mathbf{24}}$  heavy  $\mathbf{24}$  Higgs mass,  $M_{\mathbf{75}}$  heavy  $\mathbf{75}$  Higgs mass and  $M'$  mass of orthogonal components to Nambu-Goldstone multiplets. The corrections  $\Delta_{1,2}$  come from a mass splitting among Higgs multiplets. In a missing partner model, it is known that  $\Delta_1$  is sizable, e.g.,  $\Delta_1 = \ln(1.7 \times 10^4)$  and it can relax a constraint from proton decay<sup>§</sup>.

Using relations (14) and (15), we can estimate the magnitude of  $\hat{M}_C$  and  $\hat{M}_U$  as follows,

$$\hat{M}_C = M_Z \cdot \left( \frac{m_{SUSY}}{M_Z} \right)^{5/6}$$

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<sup>§</sup>See the third and fourth papers in Ref.[3].

$$\begin{aligned}
& \times \exp \left( \frac{5\pi}{6} ((3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(M_Z) + 12\alpha_U^{-1}(x'_{\mathbf{24}} - x'_{\mathbf{75}})) + \Delta_1 \right) \\
& \sim 3.2 \times 10^{15} \cdot \left( \frac{m_{SUSY}}{M_Z} \right)^{5/6} \cdot \exp \left( 10\pi\alpha_U^{-1}(x'_{\mathbf{24}} - x'_{\mathbf{75}}) + \Delta_1 \right) \quad (17)
\end{aligned}$$

$$\begin{aligned}
\hat{M}_U &= M_Z \cdot \left( \frac{M_Z}{m_{SUSY}} \right)^{2/9} \\
& \times \exp \left( \frac{\pi}{18} ((5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(M_Z) + 36\alpha_U^{-1}x'_{\mathbf{75}}) - \Delta_2 \right) \\
& \sim 4.8 \times 10^{16} \cdot \left( \frac{M_Z}{m_{SUSY}} \right)^{2/9} \cdot \exp \left( 2\pi\alpha_U^{-1}x'_{\mathbf{75}} - \Delta_2 \right) \quad (18)
\end{aligned}$$

where we use experimental data of  $\alpha_i$ .

SUSY-GUT models, in general, possess dangerous dimension-five and six operators to induce a rapid proton decay [10]. The dimension-five operators due to the exchange of colored Higgs boson are suppressed only by a single power of  $\hat{M}_C$  in most cases. The present lower bound of proton decay experiment suggests that  $\hat{M}_C$  is heavier than  $O(10^{16})\text{GeV}$ . On the other hand, the dimension-six operators due to the exchange of  $X$  and  $Y$  gauge bosons are suppressed by power of  $M_V^2$ . Hence if  $M_V$  is  $O(10^{16})\text{GeV}$ , nucleon lifetime can be longer than  $10^{34}$  years. As we can see from Eqs. (17) and (18), the magnitude of  $\hat{M}_C$  and  $M_V$  is sensitive to that of  $\Delta_{1,2}$  and  $\alpha_U^{-1}x'_R$ . It is important to get values or a relation between  $\alpha_U^{-1}x'_R$  from other analysis. Then we can obtain useful information on GUT scale mass spectrum and scales such as  $\hat{M}_C$  and  $\hat{M}_U$  from precision measurement of sparticle masses and RG analysis.

In the same way, we have analyzed a non-SUSY case with gravitational correction. Under the assumption that particle contents of  $SU(5)$  GUT are  $SU(5)$  gauge boson  $\mathbf{24}$ , Higgs bosons  $\mathbf{24}$  and  $\mathbf{75}$ , matter fermions  $N_g(\mathbf{\bar{5}} + \mathbf{10})$ , a fundamental representation Higgs  $\mathbf{5}$  and extra matter fields, we get the following relations at one-loop level,

$$\begin{aligned}
& (3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(M_Z) + 12\alpha_U^{-1}(x'_{\mathbf{24}} - x'_{\mathbf{75}}) \\
& = \frac{1}{5\pi} \left( \ln \frac{\hat{M}_C}{M_Z} - \Delta_1 \right), \quad (19)
\end{aligned}$$

$$(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(M_Z) + 36\alpha_U^{-1}x'_{\mathbf{75}} = \frac{22}{\pi} \left( \ln \frac{\hat{M}_U}{M_Z} + \Delta_2 \right). \quad (20)$$

Using relations (19) and (20), we can estimate the magnitude of  $\hat{M}_C$  and  $\hat{M}_U$  as follows,

$$\begin{aligned}\hat{M}_C &= M_Z \cdot \exp \left( 5\pi(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(M_Z) + 60\pi\alpha_U^{-1}(x'_{\mathbf{24}} - x'_{\mathbf{75}}) + \Delta_1 \right) \\ &\sim 10^{83} \cdot \exp \left( 60\pi\alpha_U^{-1}(x'_{\mathbf{24}} - x'_{\mathbf{75}}) + \Delta_1 \right)\end{aligned}\quad (21)$$

$$\begin{aligned}\hat{M}_U &= M_Z \cdot \exp \left( \frac{\pi}{22}(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(M_Z) + \frac{18\pi}{11}\alpha_U^{-1}x'_{\mathbf{75}} - \Delta_2 \right) \\ &\sim 10^{14} \cdot \exp \left( \frac{18\pi}{11}\alpha_U^{-1}x'_{\mathbf{75}} - \Delta_2 \right)\end{aligned}\quad (22)$$

where we use experimental data of  $\alpha_i$ .

We require that the magnitude of  $\hat{M}_U$  is bigger than  $O(10^{16})$  to suppress a rapid proton decay by the exchange of  $X$  and  $Y$  gauge bosons. Hence the magnitude of  $x'_{\mathbf{75}}$  is estimated as  $x'_{\mathbf{75}} = 0.024 \sim 0.044$  for  $\hat{M}_U = 10^{16 \sim 18}$  GeV,  $\Delta_2 = 1$  and  $\alpha_U = 1/45$ . Further we obtain a reasonable value as  $x'_{\mathbf{75}} - x'_{\mathbf{24}} \sim 0.019$  for  $\hat{M}_C = 10^{16 \sim 18}$  GeV,  $\Delta_1 = 10$  and  $\alpha_U = 1/45$ . Hence the non-SUSY  $SU(5)$  GUT can revive in the presence of generic gravitational corrections.

5. To summarize, we have studied non-universal corrections to gauge couplings due to higher dimensional operators along the three independent directions, **24**, **75** and **200** directions, and their linear combinations based on the SUSY  $SU(5)$  GUT. We have obtained a good agreement with the experimental values of  $\alpha_i$  with  $|x| \lesssim O(0.01)$  and  $M_U = O(10^{16.1 \sim 16.2})$  GeV for the three pure directions. A higher energy scale can be allowed as a breaking scale of  $SU(5)$  in the presence of gravitational corrections by a certain linear combination of contribution from **24** and **75** Higgs fields. The constraints from the suppression of rapid proton decay is sensitive to magnitude of  $\Delta_{1,2}$  and  $\alpha_U^{-1}x'_R$ . It is important to get values or a relation between  $\alpha_U^{-1}x'_R$  from other analysis. Then we can obtain useful information on GUT scale mass spectrum and scales such as  $\hat{M}_C$  and  $\hat{M}_U$  from precision measurement of sparticle masses and RG analysis. The non-SUSY  $SU(5)$  GUT can be revived in the presence of gravitational corrections.

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